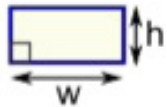


Geometry Handout

Rectangle and Square

Area of a Rectangle and Square (square has all sides equal)



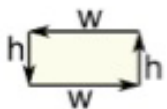
The Area is the width times the height:
Area = $w \times h$

Example: A rectangle is 6 m wide and 3 m high; what is its Area?

$$\text{Area} = 6 \text{ m} \times 3 \text{ m} = 18 \text{ m}^2$$

Perimeter of a Rectangle

The Perimeter is the distance around the edges.



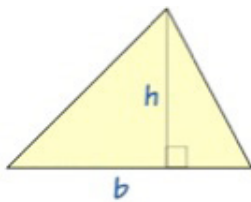
The Perimeter is 2 times the (width + height):
Perimeter = $2(w+h)$

Example: A rectangle has a width of 12 cm and a height of 5 cm; what is its Perimeter?

$$\text{Perimeter} = 2 \times (12 \text{ cm} + 5 \text{ cm}) = 2 \times 17 \text{ cm} = 34 \text{ cm}$$

Triangle

Area of a Triangle



The Area is half of the base times height.

- "b" is the distance along the base
- "h" is the height (measured at right angles to the base)

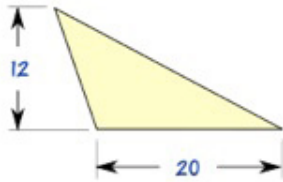
$$\text{Area} = \frac{1}{2} \times b \times h$$

The formula works for all triangles.

Note: Another way of writing the formula is **bh/2**

Example: What is the Area of this triangle?

(Note: 12 is the **height**, not the length of the left-hand side)



Height = $h = 12$

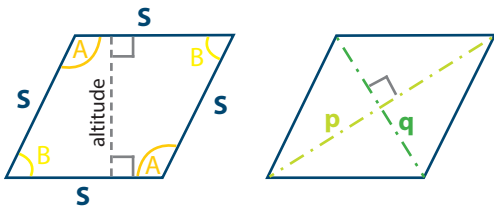
Base = $b = 20$

Area = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 20 \times 12 = 120$

Perimeter of a Triangle

The Perimeter is the distance around the edge of the triangle; just add up the three sides.

Rhombus

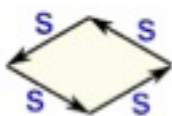


Area of a Rhombus

The Area can be calculated by:
the altitude times the side length:
Area = altitude \times s

Perimeter of a Rhombus

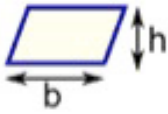
The Perimeter is the distance around the edges.



The Perimeter is **4 times "s" (the side length)**
because all sides are equal in length:
Perimeter = $4s$

Parallelogram

Area of a Parallelogram



The Area is the base times the height:

$$\text{Area} = b \times h$$

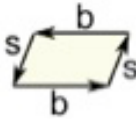
(h is at right angles to b)

Example: A parallelogram has a base of 6 m and is 3 m high; what is its Area?

$$\text{Area} = 6 \text{ m} \times 3 \text{ m} = 18 \text{ m}^2$$

Perimeter of a Parallelogram

The Perimeter is the distance around the edges.



The Perimeter is 2 times the (base + side length):

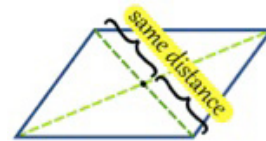
$$\text{Perimeter} = 2(b+s)$$

Example: A parallelogram has a base of 12 cm and a side length of 6 cm; what is its Perimeter?

$$\text{Perimeter} = 2 \times (12 \text{ cm} + 6 \text{ cm}) = 2 \times 18 \text{ cm} = 36 \text{ cm}$$

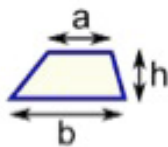
Diagonals of a Parallelogram:

The diagonals of a parallelogram bisect each other. In other words, the diagonals intersect each other at the half-way point.



Trapezoid

Area of a Trapezoid



The Area is the **average of the two base lengths** times the **altitude**:

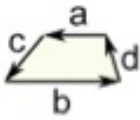
$$\text{Area} = 1/2(a+b) \times h$$

Example: A trapezoid's two bases are 6 m and 4 m, and it is 3 m high. What is its Area?

$$\text{Area} = 1/2(6 \text{ m} + 4 \text{ m}) \times 3 \text{ m} = 5 \text{ m} \times 3 \text{ m} = \mathbf{15 \text{ m}^2}$$

Perimeter of a Trapezoid

The Perimeter is the distance around the edges.



The Perimeter is **the sum of all side lengths**:

$$\text{Perimeter} = a + b + c + d$$

Example: A trapezoid has side lengths of 5 cm, 12 cm, 4 cm, and 15 cm. What is its Perimeter?

$$\text{Perimeter} = 5 \text{ cm} + 12 \text{ cm} + 4 \text{ cm} + 15 \text{ cm} = \mathbf{36 \text{ cm}}$$

Circle



$$\frac{\text{Circumference}}{\text{Diameter}} = \pi = 3.14159\dots$$

The **Radius** is the distance from the center to the edge.

The **Diameter** starts at one side of the circle, goes through the center, and ends on the other side.

The **Circumference** is the distance around the edge of the circle.

And here is the really cool thing:

When you divide the circumference by the diameter you get 3.141592654..., which is the number π (Pi).

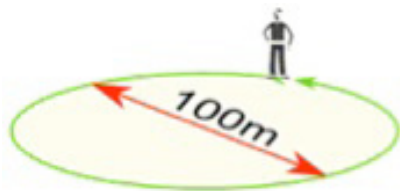
So when the diameter is 1, the circumference is 3.141592654...



We can say:

$$\text{Circumference} = \pi \times \text{Diameter}$$

Example: You walk around a circle which has a diameter of 100 m; how far have you walked?



$$\begin{aligned} \text{Distance walked} &= \text{Circumference} = \pi \times 100 \text{ m} \\ &= \mathbf{314 \text{ m}} \text{ (to the nearest m)} \end{aligned}$$

Also note that the Diameter is twice the Radius:

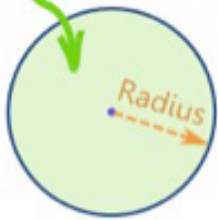
$$\text{Diameter} = 2 \times \text{Radius}$$

And so this is also true:

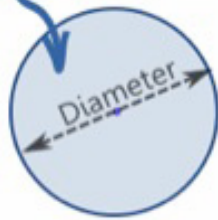
$$\text{Circumference} = 2 \times \pi \times \text{Radius}$$

Area of a Circle

$$\text{Area} = \pi \times \text{radius}^2$$

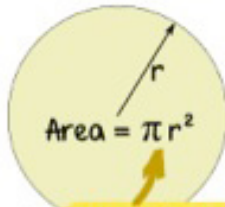


$$\text{Area} = (\pi/4) \times \text{Diameter}^2$$



The Area of a circle is π times the Radius squared, which is written:

$$A = \pi r^2$$



"Pie Are Squared"
... but they are round!

To help you remember, think "Pie Are Squared" (even though pies are usually round).

Or, in relation to Diameter:

$$A = (\pi/4) \times D^2$$

Example: What is the area of a circle with radius of 1.2 m?

$$A = \pi \times r^2$$

$$A = \pi \times 1.2^2$$

$$A = \pi \times (1.2 \times 1.2)$$

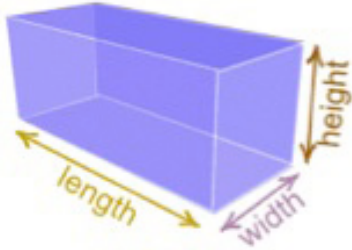
$$A = 3.14159... \times 1.44 = 4.52 \text{ (to 2 decimals)}$$

Cuboid or Cube

Volume of a Cuboid

A cuboid is a 3-dimensional shape.

So to work out the volume we need to know 3 measurements.



Look at this shape.

There are 3 different measurements:

Length, Width, Height

The volume is found using the formula:

Volume = Length \times Width \times Height

Which is usually shortened to:

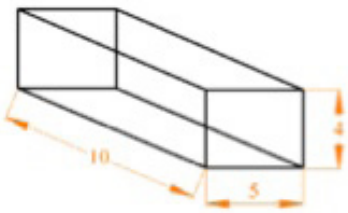
$$V = l \times w \times h$$

Or more simply:

$$V = lwh \text{ (in any order)}$$

It doesn't really matter which one is length, width, or height, so long as you multiply all three together.

Example: Lengths in meters (m):



The volume is:

$$10 \text{ m} \times 5 \text{ m} \times 4 \text{ m} = \mathbf{200 \text{ m}^3}$$

It also works out the same like this:

$$4 \text{ m} \times 5 \text{ m} \times 10 \text{ m} = 200 \text{ m}^3$$

Note: the result is in m^3 (cubic meters) because we have multiplied meters together three times.

Perimeter

The Perimeter of a cuboid equals:

$$2(\text{length} \times \text{width}) + 2(\text{length} \times \text{height}) + 2(\text{width} \times \text{height})$$

Example problem from diagram shown above:

$$\text{Perimeter} = 2(10 \text{ m} \times 5 \text{ m}) + 2(10 \text{ m} \times 4 \text{ m}) + 2(5 \text{ m} \times 4 \text{ m})$$

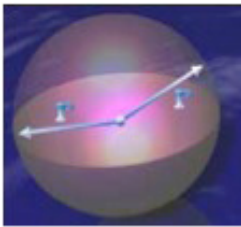
$$\text{Perimeter} = 100 \text{ m}^2 + 80 \text{ m}^2 + 40 \text{ m}^2 = 220 \text{ m}^2$$

Sphere

Sphere Facts

Notice these interesting things:

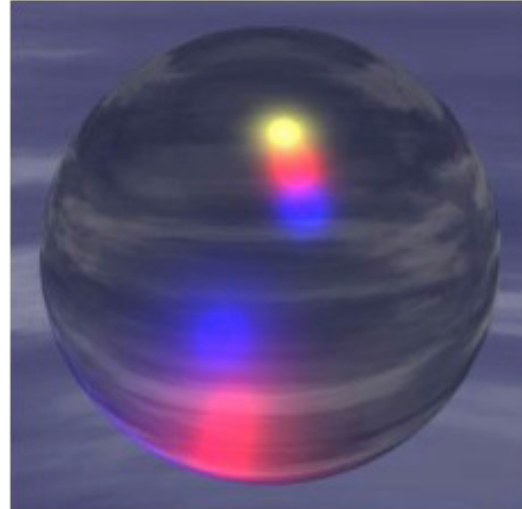
- It is perfectly symmetrical
- It has **no** edges or vertices (corners)
- It is **not** a polyhedron
- All points on the surface are the same distance from the center



And for reference:

$$\text{Surface Area} = 4 \times \pi \times r^2$$

$$\text{Volume} = (4/3) \times \pi \times r^3$$



Glass Sphere

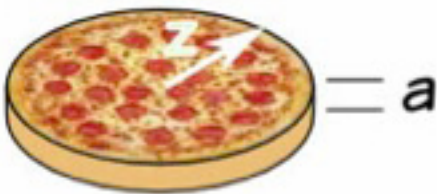
Balls and marbles are shaped like spheres.

Cylinder

Just multiply the area of the circle by the height of the cylinder:

- Area of the circle: $\pi \times r^2$
- Height: h
- Volume = Area \times Height = $\pi \times r^2 \times h$

How to remember: Volume = pizza



Imagine you just cooked a pizza.

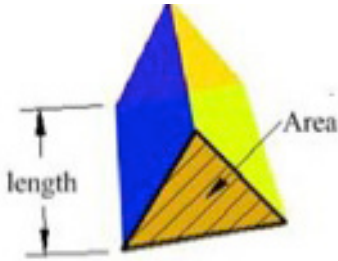
The radius is "z", and the thickness "a" is the same everywhere ... What is the volume?

Answer: $\pi \times z \times z \times a$

(We would normally write "pi" as π , and $z \times z$ as z^2 , but you get the idea!)

Prism

The Volume of a prism is the area of one end times the length of the prism.



$$\text{Volume} = \text{Area} \times \text{Length}$$

Example: What is the volume of a prism whose ends have an area of 25 m^2 and that is 12 m long?

$$\text{Answer: Volume} = 25 \text{ m}^2 \times 12 \text{ m} = 300 \text{ m}^3$$

Cone



Volume of a Cone vs. Cylinder

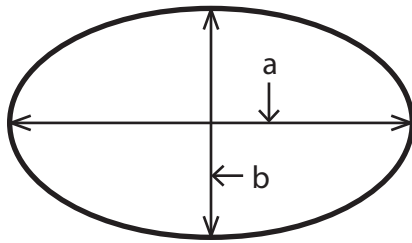
The volume formulas for cones and cylinders are very similar:

The volume of a cylinder is: $\pi \times r^2 \times h$

The volume of a cone is: $\pi \times r^2 \times (h/3)$

So, the only difference is that a cone's volume is one-third ($1/3$) of a cylinder.

Ellipses



$a = 1/2$ major axis

$b = 1/2$ minor axis

Area

$$\text{Area} = \pi ab$$

$$a = 1/2 \text{ the major axis} \quad b = 1/2 \text{ the minor axis}$$

Example: Find the Area of an elliptical swimming pool whose major axis is 9 meters and minor axis is 6 meters.

$$a = 0.5(9 \text{ m}) = 4.5 \text{ m}$$

$$b = 0.5(6 \text{ m}) = 3 \text{ m}$$

$$\text{Area} = 3.1416(4.5 \text{ m})(3 \text{ m}) = 42.4 \text{ m}^2$$